www.iop.org/Journals/PhysEd

Looking for consistency in the construction and use of Feynman diagrams

Peter Dunne

Preston College, Fulwood Campus, Preston PR2 8UR, UK

Abstract
This article describes the role of the Feynman diagram in the representation of particle interactions. Conventions for the construction and interpretation of the diagrams are summarized and examples of the applications of those conventions are given. The article draws attention to the range of Feynman diagrams in current particle physics literature and argues for a move towards consistency in their use.

Introduction

Feynman diagrams have been used over the last 50 years as a tool to illustrate and describe particle interactions. They show the participating particles and indicate the interaction types.

The diagrams were originally developed by Richard Feynman, in the late 1940s and early 1950s, as informal shorthand sketches to keep a tally of particle interactions. Feynman was building the theory that became known as quantum electrodynamics, a powerful mathematical theory of interactions between photons and other particles. His sketch diagrams were used to illustrate aspects of the mathematics. They showed the numbers of ways that photons could be emitted and absorbed by interacting particles.

The use of the diagrams might be compared to the way we use triangles or polygons of vectors to illustrate equilibrium of forces. There are rules for the construction of these diagrams. For example, a closed polygon is associated with forces in equilibrium; the lengths of the sides of the polygon represent the sizes of the forces etc. The combinations of ordinary forces can be described purely mathematically but physicists recognize the value and economy of the vector diagrams in visualizing what is going on.

Feynman's diagrammatic approach to describing particle interactions was adopted by the physics community and developed beyond the simple representation of photon exchanges in quantum electrodynamics. It came to be used to represent any subatomic interaction that involved the exchange of an interaction particle. But the partial informality of the original approach led to a growth of different ways of representing the particles and their interactions.

When the Feynman diagram is combined with the framework provided by the standard model of particle physics it produces a way of checking the types of particle that must enter, or emerge from, a reaction. These useful aspects of the approach can be seen only if basic rules are applied in the construction of the diagrams. Basic construction rules can be found in some comprehensive university level textbooks [1, 2].

The Feynman diagram is becoming a regular syllabus item on new advanced level physics courses. Unfortunately, a wide range of textbooks and teacher guidance relating to particle physics does not show a consistent approach to the construction and use of the diagrams. The existence of these inconsistencies is not helpful to students who are taking their first steps into the

 0031-9120/01/050366+09$30.00

Table 1. The six quarks and six leptons.

	Charge in units of electron charge	First generation	Second generation	Third generation
The quarks	+2/3	up (u)	charm (c)	top (t)
	−1/3	down (d)	strange (s)	bottom (b)
The leptons	0	electron neutrino (ν_e)	muon neutrino (ν_μ)	tau neutrino (ν_τ)
	−1	electron (e^-)	negative muon (μ^-)	tau (τ^-)

topic of particle physics.

The purpose of this article is an attempt to persuade the teaching community to become aware of the rules of construction of the diagrams and adopt a consistent approach to the representation of particle reactions.

The standard model

The standard model of particle physics describes subnuclear processes in terms of three groups of particles. The groups are quarks, leptons and interaction or exchange particles. Quarks combine with each other to make matter particles like protons and neutrons. Leptons are particles like electrons. Leptons do not combine with each other to make other particles. Interaction or exchange particles carry information, electric charge, momentum etc between quarks and leptons, quarks and quarks or leptons and leptons.

The standard model states that there are six quark types and six lepton types. The different types of particle are light-heartedly referred to as ‘flavours’, i.e. there are six quark flavours and six lepton flavours. There are four types of interaction particle but only three of them are used in particle physics, the fourth one being associated with gravity.

The quarks and leptons are classified according to their electric charge and rest mass. They are generally arranged in groups of two called ‘generations’. The groupings are shown in table 1. Particles of the first generation have the smallest rest mass, those of the second generation have a greater rest mass and those of the third generation have the greatest rest mass. The masses of the quarks increase in the order u, d, s, c, b, t. The masses of the leptons increase in the sequence e, μ, τ. It has not yet been firmly established whether the neutrinos have mass.

Each particle has an antiparticle with opposite electric charge. The antiquarks and antileptons can be arranged in a way similar to that in table 1 but with the sign of the electric charge reversed. In the case of the uncharged particles, the antiparticles still exist but their antiparticle character is expressed in ways other than by electric charge.

The wide range of particles observed in accelerator experiments can be reduced to just leptons and two types of quark combinations. Groups of three quarks form the baryon class of particles. For example, the familiar proton and neutron are baryons with the quark combinations uud and udd respectively.

Quark–antiquark pairings form the meson class of particles. The pions and kaons, discovered in cosmic ray interactions in the late 1940s, are mesons. The positive pion is a combination of an up quark and an antidown quark. The negative kaon is the quark combination of an antiup with a strange quark. Interactions between particles can now be seen in terms of interactions between quarks and leptons.

The quark structure of some common baryons and mesons is shown in table 2.

Interactions and exchange particles

The grouping of particles illustrated in table 1 is directly relevant to the construction and interpretation of Feynman diagrams. In the case of leptons, the arms of Feynman diagrams represent particle changes within generations. The changes occur vertically within the table. For example, an electron neutrino interacts with an exchange

Table 2. Commonly occurring baryons and mesons. Note: the bars over the quark symbols indicate antiparticles.

	Name	Symbol	Quark structure
Common baryons	proton	p	uud
	neutron	n	udd
Common mesons	pi-zero (neutral pion)	π^0	$u\bar{u}$ or $d\bar{d}$
	pi-plus (positive pion)	π^+	$u\bar{d}$
	pi-minus (negative pion)	π^-	$d\bar{u}$
	K-zero (neutral kaon)	K^0	$d\bar{s}$
		$\bar{K}^0$	$\bar{d}s$
	K-plus (positive kaon)	K^+	$u\bar{s}$
	K-minus (negative kaon)	K^-	$s\bar{u}$

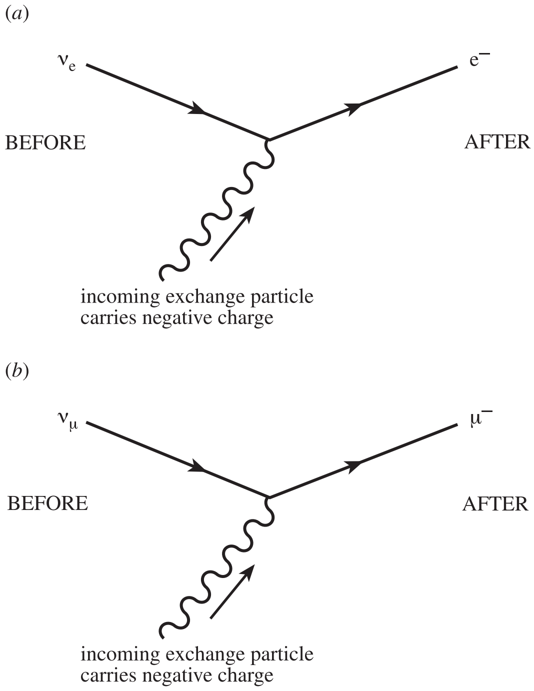


Figure 1. Feynman diagram vertices representing lepton changes within their generations. (*a*) An electron neutrino interacts with an exchange particle and becomes an electron. (*b*) A muon neutrino interacts with an exchange particle and becomes a muon.

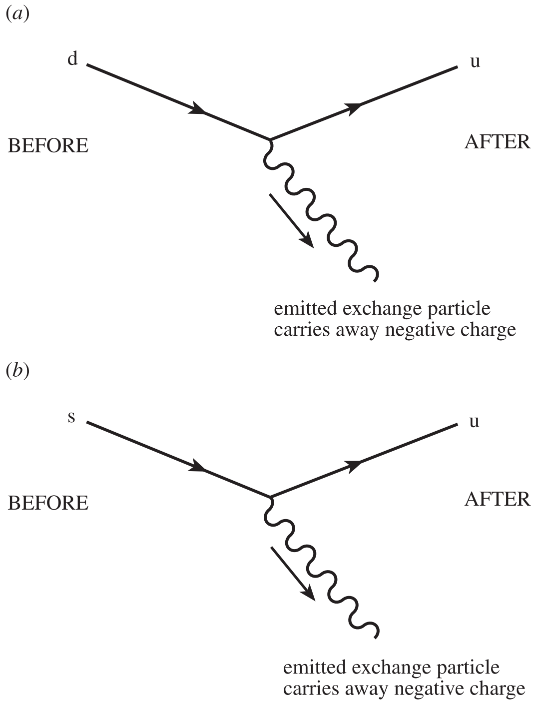


Figure 2. Feynman diagram vertices for quark changes. (*a*) A down quark emits an exchange particle to become an up quark. (*b*) A strange quark emits an exchange particle and becomes an up quark. This is a 'diagonal' flavour change where a quark transforms into a member of a neighbouring generation.

particle and becomes an electron, a muon neutrino interacts with an exchange particle and becomes a muon etc. These interactions are represented in figure 1. The representations are single Feynman diagram vertices.

In the case of quarks, changes also occur within a generation but can, in addition, occur diagonally between two generations. For example, a down quark might emit an exchange particle and become an up quark. Similarly, a strange quark might emit an exchange particle and become an up quark. These vertices for these interactions are shown in figure 2.

The interaction, or exchange, particles are the photon, the gluon, the W particle and the Z particle. The emission or absorption of exchange particles reflects the three basic types of particle interaction. The electromagnetic interaction involves the emission or absorption of photons. The strong interaction involves the emission or absorption of gluons. The weak interaction involves the emission or absorption of the W and Z particles.

The W particle carries either a positive or a negative electric charge between particles whereas the Z particle carries no electric charge. The W particle is a quark-changing or a lepton-changing particle. Gluons and photons carry no electric charge and do not change the particle flavour.

Description of the interactions in words or equations makes the subject seem very complicated and difficult to follow. The use of Feynman diagrams makes it much easier to both categorize and visualize what is going on.

Vertices

The fundamental building block of the Feynman diagram is the vertex. It can be shown [1, 2] that there is a limited set of vertices to describe the three types of interaction. Particle interactions are represented by combinations of vertices similar to those shown in figures 1 and 2. The vertices all have the same basic structure. A quark or lepton enters a vertex, emits or absorbs an exchange particle, and a related quark or lepton then emerges from the vertex. The process is either quark–vertex–quark or lepton–vertex–lepton. The fact that a particle enters a vertex, interacts with an exchange particle and then leaves a vertex is seen in the directions of the arrows drawn on the diagram. There will always be an arrow entering the vertex and one leaving it.

The vertices show the time ordering of the processes. It is important to recognize that the vertices show time orderings only; they do not represent tracks of particles in space. The convention is that the left of the diagram shows the particles before the reaction and the right of the diagram shows the particles after the reaction. The flow of time is from left to right. The orientation of the wavy line representing the exchange particle with respect to the time direction indicates whether the exchange particle is entering the vertex or leaving it.

Alternative representations have the flow of time up the diagram with the bottom representing the 'before' state and the top representing the 'after' state. Both representations are equally valid but there seems to be a preference amongst particle physicists for the left to right approach.

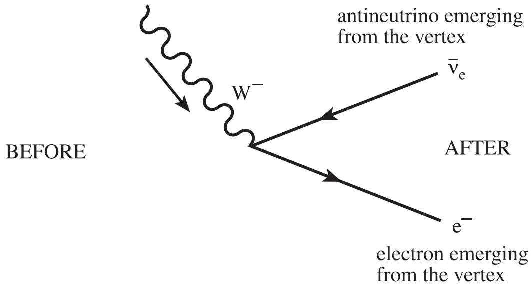


Figure 3. Particle–antiparticle pair creation. An exchange particle materializes as an electron and an electron antineutrino. One of the arrows must point backwards towards the vertex.

Labels are placed at the ends of the arrows to indicate particles that are seen in the real world. In the case of a particle–antiparticle pair emerging from a reaction, one of the arrows needs to point 'backwards' towards the vertex in order to fulfil the requirement of a particle entering the vertex followed by one of the same class leaving the vertex. This situation is illustrated in figure 3.

It is this backward placement of the arrow that led to the view of an antiparticle being a real particle travelling backwards in time. Whilst the suggestion of something travelling backwards in time excites the imagination of some students it is preferable to say that a backwards arrow simply represents an antiparticle travelling forward in time and that the direction of the arrow is chosen simply to ensure that the vertex rules are obeyed. At the vertex itself, a lepton enters and a related lepton leaves but the symbols placed at the ends of the arrows indicate the particle and antiparticle pair that must emerge from the reaction.

Using this arrow direction convention, the vertex for an antiparticle entering a reaction and then leaving it would have the form shown in figure 4. Particle–antiparticle annihilation would be represented as in figure 5.

Common Feynman diagrams

The conventions for drawing Feynman diagrams are described in box 1. If the conventions are used then Feynman diagrams can be constructed from combinations of vertices to work out

Box 1. Conventions for drawing Feynman diagrams

1. Arrowheads

Feynman diagrams are combinations of vertices. Each vertex has an arrowhead pointing towards the vertex and an arrowhead leaving it. This represents a same-generation lepton–lepton transition or a same- or neighbouring-generation quark–quark transition. The arrowheads are drawn in the lines rather than at the end of them.

2. Lines

The lines representing the quarks or leptons are straight solid lines. The lines representing the exchange particles are wavy in the case of photons, W particles or Z particles and curly in the case of gluons. Some authors use broken straight lines to represent the W and Z particles. Whilst it is not intentional, the common use of the same wavy line to represent both the weak interaction particles and the photon reflects the fact the theories of the electromagnetic interaction and the weak interaction have now been combined into one electroweak theory.

3. Time direction

The diagram shows time flow with time going from left to right (or from bottom to top). Arrows drawn in the same direction as the increasing time direction represent particles travelling forward in time. Arrows drawn against the time direction represent antiparticles travelling forward in time.

4. Particle labels

Particle labels such as e^-, u, d etc are placed outside the diagram at the ends of the arrow lines. The labels represent free particles approaching or leaving a reaction. If the arrowheads point backwards in time then the free particle must be an antiparticle and be labelled as such.

5. Exchange particles

Exchange particles link vertices. Arrowheads are not normally drawn on exchange particle lines. The direction of the exchange particle flow is indicated by the line's orientation on the diagram. The line is labelled with the symbol for the exchange particle being drawn alongside the lines.

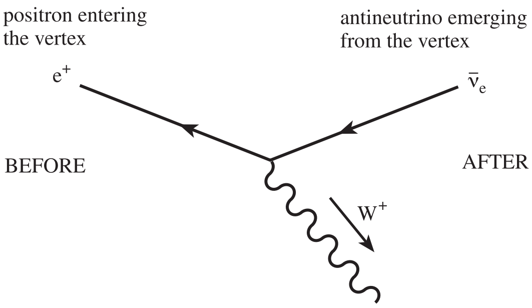


Figure 4. A positron enters a vertex, emits an exchange particle and emerges as an antineutrino.

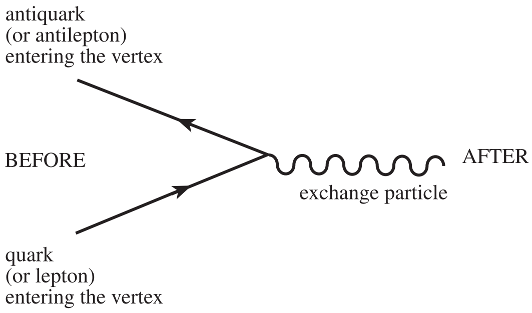


Figure 5. Particle–antiparticle annihilation seen as a simple vertex with an exchange particle emerging.

whether interacting particles are real particles, antiparticles, quarks, leptons etc. For example, if two particles emerge from a vertex then they must be a particle–antiparticle pair of the same type in order to obey the vertex rules.

Most of the illustrations presented as

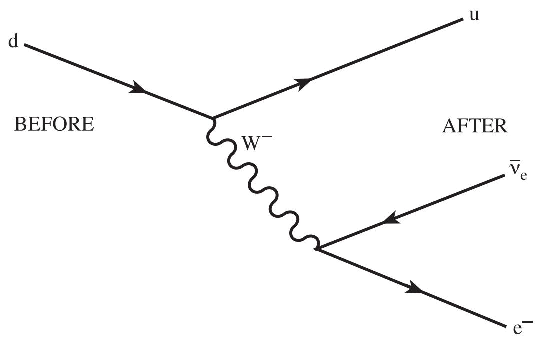


Figure 6. The standard beta decay diagram. This is a combination of a quark–quark vertex and a lepton–lepton vertex.

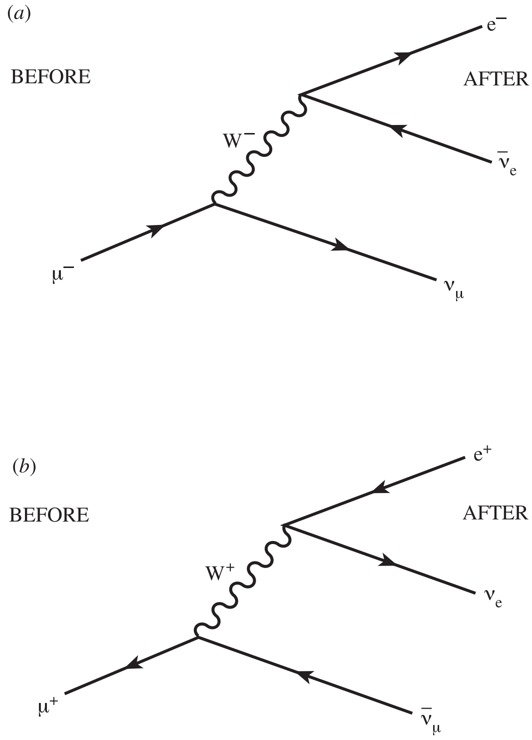


Figure 7. (*a*) The Feynman diagram for ordinary negative muon decay. (*b*) The diagram for positive muon decay.

Feynman diagrams in standard textbooks and A-level exam papers do not follow these conventions nor do they appear to follow any convention. Students working with a range of sources can see the same reaction presented in a number of different ways.

Common diagrams used as examples in introductory texts are ones to represent beta decay and negative muon decay. In beta decay, a neutron changes into a proton with the emission of an electron and an electron antineutrino, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$. At the quark level the process involves a down quark changing into an up quark. This example of a quark–vertex–quark transition is shown in figure 6. A down quark enters a vertex and leaves as an up quark. The exchange particle emitted at the vertex must carry away negative charge. It is a weak interaction so the particle is a W^- particle. The W^- particle then decays, producing a lepton vertex. One arrow must enter the vertex and one must leave so the particles emerging must be a particle–antiparticle pair. The exchange particle is negatively charged so the emerging lepton is a negative electron and the antilepton must be an electron antineutrino because the leptons must come from the same generation.

It is worth noting that this is the preferred particle combination in ordinary beta decay but if the original down quark had enough energy then the W^- particle could equally materialize into a negative muon and its associated antineutrino, or a down quark and an antiup quark.

Muon decay is an example of a lepton–vertex–lepton process. In figure 7(*a*) a negative muon emits a W^- particle and emerges as a muon neutrino. The W^- particle materializes into a particle–antiparticle pair as required by the vertex rule. In this example the pair consists of a negative electron and the electron antineutrino.

In figure 7(*b*) a positive muon emits a W^+ and emerges as a muon antineutrino. The backward facing arrows represent the antiparticle entering the vertex and the related antiparticle leaving it. The emitted exchange particle materializes as a positron and an electron neutrino with the arrow directions indicating which is the particle and which is the antiparticle.

Figures 8(*a*) and 8(*b*) are derived from figure 7(*a*) by swinging the arrows around the vertices. Using figure 7(*a*) and swinging the electron antineutrino arrow around the vertex produces figure 8(*a*), in which an electron neutrino interacts with a negative muon with the emergence of a muon neutrino and a negative electron.

Alternatively, using figure 7(*a*) and swinging the negative electron arrow around to the 'before' side produces a backward facing arrow to give figure 8(*b*). The new diagram now represents

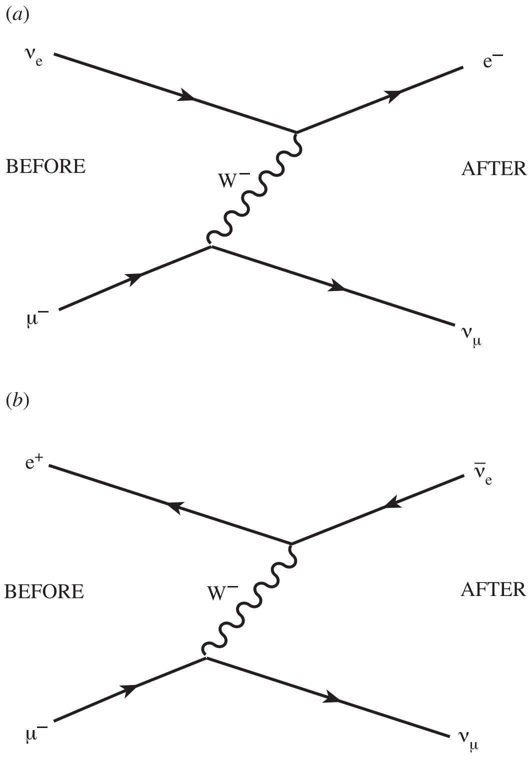


Figure 8. (*a*) An electron neutrino interacts with a muon to produce an electron and a muon neutrino. The diagram is derived directly from figure 7(*a*). (*b*) A further interaction derived from figure 7(*a*). In this case a positron–muon interaction results in the emergence of a muon neutrino and an electron antineutrino.

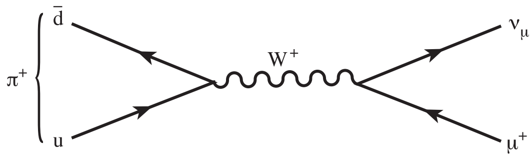


Figure 9. Pion decay. The quark–antiquark pair annihilate to produce an exchange particle. The exchange particle then materializes into a lepton–antilepton pair.

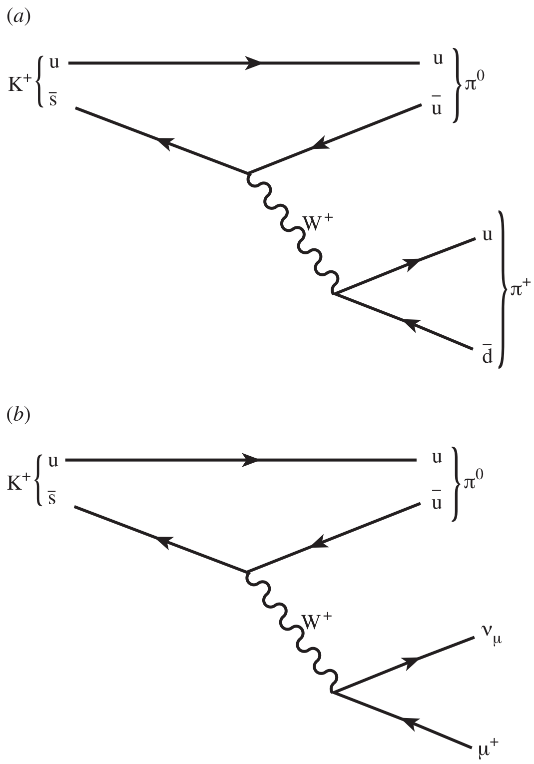


Figure 10. (*a*) Kaon decay into a pair of pions seen at the quark level. (*b*) An alternative kaon decay path into a positive muon and its neutrino.

a positron interacting with a negative muon and emerging as a muon neutrino and an electron antineutrino.

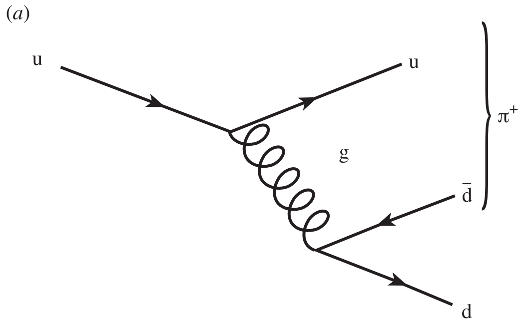
The reactions described in figures 7 and 8 are possible processes that can be seen to be different aspects of the same set of vertices. Any consistent Feynman diagram can have the vertices swung round to produce alternative related reactions.

More examples

Figure 9 represents the decay of a pion into a positive muon and a muon neutrino, i.e. $\pi^+ \rightarrow \mu^+ + \nu_\mu$. The up quark and the antidown quark annihilate to produce a W^+ particle. This annihilation can be seen as a quark–vertex–quark transition with a backward facing arrow representing the antiquark in the pion. The emergent W^+ then decays into a positive muon and a muon neutrino. The arrow convention at the vertex indicates which is the particle and which is the antiparticle.

Figure 10(*a*) represents the weak decay of a positive kaon into two pions, i.e. $K^+ \rightarrow \pi^+ + \pi^0$. The antistrange quark in the kaon undergoes a 'diagonal' change into an antiup quark with the emission of a W^+. The W^+ materializes into a quark–antiquark pair and the resulting quarks recombine to produce two pions.

Figure 10(*b*) represents a different allowed outcome from the same starting point. In this case the kaon decays into a neutral pion, a positive muon and a muon neutrino.



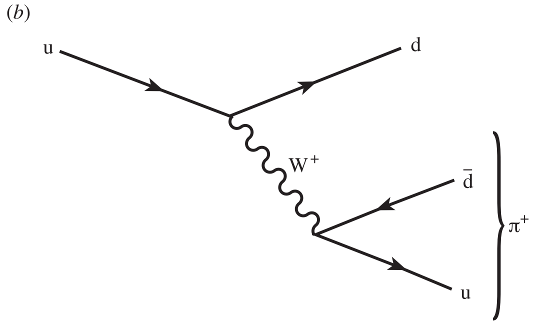


Figure 11. (*a*) Pion production in a proton–proton collision. A gluon is released from a quark and then materializes into a same flavour quark–antiquark pair. (*b*) An alternative outcome from a proton–proton collision in which a quark changes flavour. The overall outcome is the same as that in (*a*).

The reader might note that the combination of the diagram in figure 9 with the outcomes of figure 10(*a*) can produce the outcomes indicated in figure 10(*b*); consistent Feynman diagrams can be added to each other.

Two final examples of vertex combinations are worth looking at. Figure 11(*a*) illustrates pion production in proton–proton collisions. The reaction is $p + p \rightarrow p + n + \pi^+$. This can be interpreted as a strong interaction in which a quark from one of the protons releases a gluon, which then materializes into a quark–antiquark pair. The gluon is uncharged so the quark–vertex–quark arrangement at the gluon decay must represent a particle–antiparticle pair of the same flavour to ensure that the charges cancel out. The emerging quarks reshuffle with the up quark combining with the antidown quark to become the positive pion. The remaining d quark combines with the u and d from the original proton to form a neutron.

An alternative interpretation, given in figure 11(*b*), is that an up quark in the proton emits a W^+ particle, which then materializes into an up and an antidown quark. In this case, the exchange particle is charged and so the emerging quark–antiquark pair must be of different flavour to produce the resulting positive charge. Figure 11(*a*) is an example of a strong interaction whereas figure 11(*b*) is an example of a weak interaction that produces the same outcome. Of the two routes described, the strong interaction is going to be the dominant one. This is due to the fact that the range of the strong interaction is about 10^{-15} m, in contrast to the weak interaction which has a range of about 10^{-18} m. In the p–p collision the protons are going to feel the strong interaction well before they might feel the weak interaction, so the process suggested by figure 11(*a*) is going to be the preferred one.

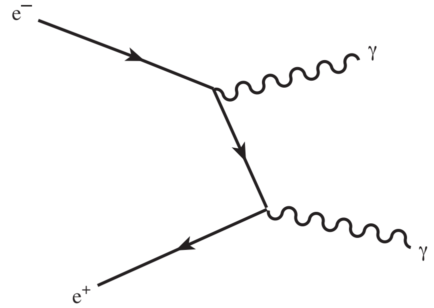


Figure 12. Electron–positron annihilation seen as the emission of two photons at two successive vertices.

Figure 12 is the Feynman diagram for electron–positron annihilation with the emergence of two photons, i.e. $e^- + e^+ \rightarrow \gamma + \gamma$. In this case a lepton passes through two vertices with the emission of a photon at each vertex. The photons emerge as real particles.

Conclusion

Feynman diagrams can be used as a valuable interpretative tool if certain construction conventions are applied. The diagrams should be seen as simple combinations of basic vertices that represent changes within and between generations of particles in the standard model of particle physics.

Unfortunately a wide range of textbooks and teacher guidance has presented the Feynman diagram as an informal illustration rather than a formal tool. These informal representations follow no real conventions and astute students working from a range of sources can be left a little bewildered by the diagram differences and apparent contradictions.

In contrast, some good quality university-level texts describe clear conventions to be used to ensure the construction of meaningful and predictive diagrams to describe particle interactions. This article has shown a range of examples of such diagrams to demonstrate the application of the conventions. It has also related those conventions to the tables of particles associated with the standard model of particle physics.

It is suggested that there is a need to encourage authors, teachers and students to move away from the informal approach and adopt consistent rules when presenting Feynman diagrams.

Received 11 April 2001
PII: S0031-9120(01)23837-4

References

[1] Martin B R and Shaw G 1995 *Particle Physics* (Chichester: Wiley)
[2] Williams W S C 1997 *Nuclear and Particle Physics* (Oxford: Clarendon)

Peter Dunne graduated from the University of Durham in 1976 and gained an MSc from the University of York in 1989. He is a lecturer in physics at Preston College and his main interests are particle physics, electronics and computing. He runs extracurricular projects involving cosmic ray measurements.